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# Defining and Evaluating the Capacity Value of Distributed Generation

C.J. Dent, *Senior Member, IEEE*, A. Hernandez-Ortiz, S.R. Blake, D. Miller, D. Roberts

**Abstract**—Installed capacities of distributed generation are projected to increase substantially in Great Britain and many other power systems. This paper will discuss the definition of capacity value of DG arising from its ability to support additional demand without the need for new network capacity, in analogy with the definition of Effective Load Carrying Capability (ELCC) at transmission level. This calculated ELCC depends on the precise detail of its definition; in particular in a demand group fed by a pair of circuits where the double outage state dominates the calculated reliability index, the ELCC will be very small unless the generator can run in islanded mode. Finally, requirements for use in practical planning studies and development of formal planning standards will be discussed.

**Keywords** - power system planning; power system operation; power system reliability; risk analysis; wind energy

## I. INTRODUCTION

INSTALLED capacities of distributed generation (DG), i.e. generation embedded in distribution networks, are projected to increase substantially in Great Britain (GB) and many other power systems. This is largely due to incentives to encourage the uptake of low carbon technologies at all voltage levels of the power system, from domestic properties to higher distribution voltages. General surveys of methods for analysis of the consequences of installing DG may be found in [1], [2].

One key benefit which DG potentially brings in distribution networks is reduction in the incoming circuit capacity required from higher voltage levels to the demand group containing the DG, deferring upgrades which are driven by load growth. This has been studied by various approaches, including formal optimisation methods for network design (see [3] and references therein), and more detailed models of network reliability with Monte Carlo simulation of model outputs (e.g. [4], [5]). Discussion from an industry perspective of equivalence from a reliability perspective of generators and circuit upgrades may be found in [6]. Related work on use-of-system pricing based on contribution to deferring upgrades and reliability may be found in [7], [8].

It is widely accepted that the only systematic framework in which DG's contribution to demand security can be assessed is probabilistic risk modelling. This recognises the random nature

of outages, and provides the necessary means for considering coincident events of different natures (for instance in order to have a supply shortage, it might be necessary to have high demand, low available DG capacity, and an incoming circuit outage) and multiple resources (including incoming circuits, DG and low voltage interconnection).

The present network planning standard in Great Britain (described in Engineering Recommendation P2/6 [9] and Engineering Technical Report ETR130 [10]) states that for a demand group without DG, peak demand must be less than the incoming circuit capacity in a defined outage state; the degree of redundancy required may be N-1 and N-2 depending on the size of the demand group (N-x means that all demand must be met with any x circuits on outage.) A general survey of common mode events, which are key to the network part of this analysis, may be found in [11]. P2/6 specifies a calculation approach for determining a capacity value for DG by which incoming circuit capacity may be reduced due to its presence. The tables for DG contribution within the P2/6 standard (essentially a specified capacity value) are derived using a probabilistic calculation, but this calculation is only indirectly relevant to the network situation under study; the implications of this will be described more fully in the next section.

GB is unusual in having a formal planning standard of this form, which applies to all areas of distribution network across a whole interconnection or national system. In N America, IEEE Standard 1547 [12] indicates issues which should be considered when integrating DG, but does not specify in detail quantitative analysis approaches; in [13] the US Department of Energy investigates various aspects of integrating DG including capacity contribution. Puerto Rico also uses IEEE 1547, and consequences of DG are then analysed on a case by case basis following the assessor's judgment to quantify the capacity contribution [14]. China likewise analyses each case on an individual basis [15]. Examples of national distribution standards include the following: Australia is developing a national planning framework into which distribution providers may opt in (see p7 of [16]); Oman has a national planning standard covering multiple companies on a similar basis to GB [17]; Abu Dhabi has a standard on a very similar basis to GB [18]. Given both this broad international interest in DG, and direct relevance to specific planning standards, the subject of DG's contribution to distribution network demand security is timely.

Concepts of capacity value are well studied in the transmission-level reliability literature; a recent comprehensive survey may be found in [19]. The capacity value of an additional generator (or ensemble thereof) is made specific

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using metrics such as Effective Load Carrying Capability (ELCC, the additional load which can be supported by the additional generation without increasing the adequacy risk level), or Equivalent Firm Capacity (EFC, the completely firm generating capacity which would give the same risk level if it replaced the additional generation). It is important to note that due to the different capacity value definitions (e.g. EFC and ELCC) there can be no one definitive capacity value of a generator; however, for a given engineering question, the appropriate capacity value definition is usually clear.

Given an appropriate specification of a reliability index such as Expected Energy Not Supplied (EENS, the customary index in similar studies in GB), quantifying DG's contribution is essentially a matter of defining and calculating an appropriate capacity value metric. This paper's contributions are to show how concepts from transmission-level capacity value calculations may be adapted for these distribution-level problems with specific reference to the GB planning approach (Section II), derive analytical results which may be used to assist in interpreting these calculations (Section III), and discuss issues involved in using these methods in practical network planning (Section VII).

To illustrate the use of these capacity value definitions and analytical results, a discussion of alternative approaches to specifying the ELCC of DG is presented in Section IV based on the analytical results derived for small DG capacity; numerical examples of how the ELCC of a DG unit depends on its properties, and also the properties of the demand group and incoming network, are given in Section VI based on data described in Section V. Finally conclusions are given in Section VIII.

Such capacity value metrics are both valuable in understanding the contribution of distributed resources in any system, and also may have specific application in inclusion of distributed resources within a formal distribution network planning standard such as that in GB. In application, in addition to specifying the probability distributions of demand, incoming circuit capacity and available DG taken in isolation, it is also necessary to specify to what extent the DG can contribute in different availability states of the incoming circuits (and in different stages of any post-fault reconfiguration) as discussed in Section VII-D. It is also important to note that in practical application the computational burden of convoluting numerically the relevant probability distributions is not high, and that thus the purpose of the analytical results derived is to gain insights into drivers of results, rather than to provide a more computationally efficient approach to evaluating numerical results.

It will be seen that there is a direct analogy between the mathematical structure of the transmission and distribution adequacy problems (demand and additional resource play the same role in each, while existing generating capacity at transmission level is replaced by existing circuit capacity at distribution level). However, the discrete nature of the available incoming circuit capacity at distribution level (constructed distributions of available existing capacity at whole-system level with large numbers of units are usually quite smooth and in large systems can be well approximated by a contin-

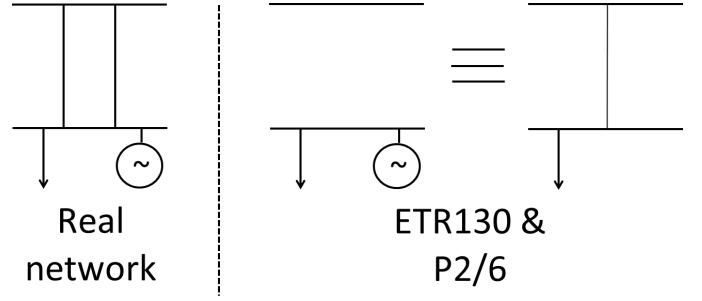


Fig. 1. Left panel: real two circuit network with N-1 security. Right panel: the cases compared in the P2/6 capacity value specification.

uous distribution), and the possible very direct relationship between available circuit capacity and the ability of distributed resources to contribute (e.g. DG might not be able to run at all in an islanded demand group without an incoming circuit to provide frequency stability) make the structure of the subsequent analysis very different. This also makes the problem of evaluating the capacity value of distributed resource distinct from that of generation capacity values in transmission-constrained systems, again due to the different form of the existing background, but also due to the more complex nature of the network constraint issues at transmission level which might not allow the analytical results and consequent insights developed here.

## II. DEFINITIONS OF CAPACITY VALUE

This section will describe the options available for defining capacity value of distributed generation, and gives a technical description of the methodology underlying the GB P2 planning standard. Finally it will conclude that Effective Load Carrying Capability (ELCC) is the most appropriate definition of capacity value for distribution planning purposes.

### A. Underlying Network Model and Analogy to Transmission

The network model used in this paper is illustrated in the left panel of Fig. 1. A demand group is supplied by incoming circuits, which have stochastic availability (with total incoming circuit capacity  $X$ ), and supply at the Grid Supply Point (GSP) is assumed to be perfectly reliable. There is also generation in the group, with available capacity  $Y$ , and the demand is  $D$ .  $X$ ,  $Y$  and  $D$  are all random variables, and we are interested in the margin of available supply over demand

$$Z = X + Y - D. \quad (1)$$

The Loss of Load Probability (LOLP) is then defined as  $I^{\text{LOLP}} = P(Z < 0)$ , and the Expected Power Not Supplied (EPNS) (or Expected Power Unserved, EPU), as  $I^{\text{EPNS}} = E[\max(-Z, 0)]$ . Adequacy is usually measured using whole season indices such as the Loss of Load Expectation (LOLE) or Expected Energy Not Supplied (EENS) in a future season under study; these are defined as

$$I^{\text{LOLE}} = \sum_t I_t^{\text{LOLP}} \quad (2)$$

$$I^{\text{EENS}} = \Delta t \sum_t I_t^{\text{EPNS}}, \quad (3)$$

where the sums are over time periods in the future season, and  $\Delta t$  is the length of time periods into which the season is divided. As described in [20], ‘whole season’ indices such as LOLE and EENS are equivalent to these snapshot indices if  $X$ ,  $Y$  and  $D$  are the available capacities and demand at a randomly chose point in time, with

$$I^{\text{LOLE}} = nI^{\text{LOLP}} \quad (4)$$

$$I^{\text{EENS}} = n(\Delta t)I^{\text{EPNS}}, \quad (5)$$

where  $n$  is the number of time periods in the future season. Throughout this paper, the snapshot picture will be used for theoretical exposition, as it simplifies the required algebraic manipulations.

The snapshot margin in Eq. (1) is specified above for a distribution demand group. The same probability model is widely used at transmission level in calculations where effects of finite network capacity are not considered ([19], [21], [22], and represented in this form in [20], [23]); in transmission,  $X$  and  $Y$  would represent available existing and additional generation, and  $D$  demand. EFC and ELCC, the most common definitions of transmission level capacity value, have been described verbally in the introduction; they may be defined mathematically with respect to EPNS as

$$E[\max(D - X, 0)] = E[\max(D + \nu_Y^{\text{ELCC}} - X - Y, 0)] \quad (6)$$

$$E[\max(D - X - Y, 0)] = E[\max(D - X - \nu_Y^{\text{EFC}}, 0)] \quad (7)$$

### B. The GB P2 Standard

For a demand group without DG, the GB distribution planning standard [9], [10] is a pure deterministic N-x standard; all demand must be supplied (following a prescribed restoration time) with either any 1 circuit (smaller groups) or 2 circuits (larger groups) on outage; if this is not the case, the DNO is in breach of its license.

If the demand group contains distributed generation (DG), then the present P2/6 standard specifies a capacity credit for the DG based on the concept of ‘Equivalent Circuit Capacity’ (ECC). With respect to EPNS, this is defined by:

$$E[\max(D - Y, 0)] = E[\max(D - \nu_Y^{\text{ECC}}, 0)]. \quad (8)$$

This is intended to represent an incoming circuit capacity which is equivalent (in demand security terms) to the generation  $Y$ ; the theoretical development underlying P2/6 may be found in [24]. Within the standard, the need for incoming network capacity (at the specified N-x redundancy level) may be reduced by  $\nu_Y^{\text{ECC}}$ ; again within the standard, ECCs of multiple generators are simply added to give a total ECC.

The relevance of ECC (as defined above) to the planning problem under study seems limited. This is illustrated in Fig. 1, in which the left panel represents the real system (in which the main incoming circuits are a key feature), and the right panel illustrates the two situations which are compared in the definition of ECC. For instance, in the two circuit case a capacity value assessment is performed conditional on the N-2 circuit state (in which the DG may well be unable to run in present networks), and is then applied in the N-1 state. There is no reason to assume that a capacity value based

on one existing supply background will be similar to that conditional on another background; this is consistent with the observation in transmission level calculations that calculated capacity values depend also on the background to which the additional supply is added (as discussed in [20], [23]), as well as the properties of the additional supply itself.

This paper will therefore explore alternative capacity value definitions which are rigorously founded in probabilistic risk calculations.

### C. Effective Load Carrying Capability at Distribution Level

The appropriate capacity value definition to use in a given situation is dictated by the particular application under study. The present standard specifies the demand level which a given installed supply capability can support, and thus (given an appropriate reliability standard) ELCC provides the appropriate definition as the additional demand which may be supplied when the DG is added, while maintaining the stated risk level. EFC does not answer such a practical engineering question, as a completely reliable supply of arbitrary capacity has no engineering interpretation (while demand may naturally be scaled continuously as it is in ELCC)<sup>1</sup>.

This paper will work mainly with ELCC defined with respect to the risk index EPNS, which is consistent with GB industrial practice, but there will be some brief discussion of results based on LOLE; this is specified in terms of the probability model in (6). Demand  $D$  has a continuous probability distribution, and both continuous and discrete  $Y$  will be considered.

## III. ANALYTICAL RESULTS

This section derives a number of closed form analytical results which will help in interpreting the numerical results presented later in the paper.

### A. Small $Y$ result

In analogy with the transmission level results in [23], it is possible to derive an analytical expression for the DG capacity value in the limit of small DG capacity  $Y$ .

*Result.* Suppose that

- Conditional on available supply  $X + Y$  being equal to  $w$  the calculated risk is  $R_D(w)$ , a function of the distribution of  $D$ ;
- $X$  takes discrete values  $\{x_i\}$ , with  $P(X = x_i) = p_i$ ;
- $Y$  takes non-negative values, and the maximum possible value of  $Y$  is small on the scale on which  $R_D(w)$  varies;
- $R_D(w)$  is once differentiable at  $w = x_i$  for all  $i$ .
- $Y_i$  is the variable  $Y$  conditional on  $X = x_i$ , with mean  $\mu_i$ .

Then the following approximate result for the ELCC of  $Y$  holds:

$$\nu_Y^{\text{ELCC}} \simeq \frac{\sum_i p_i \mu_i R'_D(x_i)}{\sum_i p_i R'_D(x_i)} \quad (9)$$

<sup>1</sup>This is in contrast to transmission, where EFC can be interpreted to a good approximation as the mean available capacity of equivalent non-100% reliable conventional generation.

*Proof for continuous Y.* The equation defining the ELCC is in this case

$$\sum_i p_i R_D(x_i) = \sum_i p_i \int dy f_{Y_i}(y) R_D(x_i + y - \nu_Y^{\text{ELCC}}). \quad (10)$$

Making a Taylor expansion about  $R_D(x_i)$ :

$$\begin{aligned} 0 &\simeq \sum_i p_i \int dy f_{Y_i}(y) (y - \nu_Y^{\text{ELCC}}) R'_D(x_i) \\ &= \sum_i p_i (\mu_i - \nu_Y^{\text{ELCC}}) R'_D(x_i). \end{aligned} \quad (11)$$

(9) is then obtained by rearrangement (a very similar proof applies in the case of discrete  $Y$ .)

1) *Interpretation: EPU-Based ELCC:* In this case,  $R_D(w)$  is the EPNS conditional on available supply being equal to  $w$ :

$$R_D(w) = \int_w^\infty dz (z - w) f_D(z) = \int_w^\infty dz (P(D > z)). \quad (12)$$

This result is obtained through integration by parts, noting that  $f_D(z) = -(d/dz)P(D > z)$ . Thus  $R'_D(w) = -P(D > w)$ , and (9) reduces to

$$\nu_Y^{\text{ELCC}} \simeq \frac{\sum_i p_i \mu_i P(D > x_i)}{\sum_i p_i P(D > x_i)}, \quad (13)$$

i.e. the ELCC is a weighted mean of the  $\{\mu_i\}$ , the weighting of  $\mu_i$  being the probability of being in circuit state  $i$  given that there is a shortfall in capacity.

2) *Further Remarks:* In the limit of very small  $Y$ , the calculated capacity value depends only on the mean of  $Y$  and not on its variability about the mean; this is because even if the output of  $Y$  is highly variable relative to its mean, if the range of  $Y$  is sufficiently small then that variability of  $Y$  will appear small compared to the variability of what is already present (in this case the demand).

In [23], a next order correction which depends on the variance of  $Y$  is derived as part of the expression for ELCC and EFC at small  $Y$ . This is only worthwhile in cases where the leading order term is precisely  $\mu_Y$ ; otherwise the algebra is sufficiently complex that no clear insights may be obtained.

The result (9) may be generalised to cases where  $Y$  and  $D$  are not independent by replacing  $\mu_i$  with  $\mu_{Y_i|D=x_i}$ , i.e. the mean of  $Y_i$  conditional on being in the critical region of capacity-demand balance.

### B. Upper Limit on ELCC

*Result.* Suppose that there is a circuit state  $i = 0$  for which  $Y_i = 0$  and  $x_0 = 0$ . Suppose also that  $R_D(z)$  is the EPU conditional on available supply being equal to  $z$ . Then an upper bound on the ELCC of  $Y$  is:

$$\nu_Y^{\text{ELCC}} \leq \frac{1}{p_0} \sum_{i \neq 0} p_i R_D(x_i). \quad (14)$$

*Proof.* (10) may be written as

$$\begin{aligned} \sum_i p_i R_D(x_i) &= \sum_i p_i E[R_D(x_i + Y_i - \nu_Y^{\text{ELCC}})] \\ &\geq p_0 (\mu_D + \nu_Y^{\text{ELCC}}), \end{aligned} \quad (15)$$

where the inequality follows from the observation that the LHS must be greater than or equal to the  $i = 0$  term in the sum as all the other terms in the sum are non-negative. The required result is then obtained by a simple rearrangement of (10).

## IV. EXAMPLES: TWO CIRCUIT SYSTEM

This section describes the two circuit system which will form the basis for most of the examples in this paper. It then explores the different options for defining ELCC using the analytical results of Section III to interpret consequences of these definitions. Several cases will be considered, with differences including choice of risk index, and whether the DG can contribute in the N-2 state. The section concludes with discussion of which of these options for specifying the ELCC should be preferred.

For the main examples, we will assume that all disconnected customers may be restored after a fault by reconfiguring the network so that they are supplied by a different route; more complex examples will be discussed briefly at the end of the paper. If there are two identical circuits of capacity  $c$ , then the (discrete) distribution of  $X$  will then be:

$$P(X = 0) = p_{N-2} \quad (16)$$

$$P(X = c) = p_{N-1} \quad (17)$$

$$P(X = 2c) = 1 - p_{N-2} - p_{N-1}. \quad (18)$$

$Y$  will usually be assumed to depend on the circuit state  $X$ ; however the case where  $Y$  is independent of  $X$  will also be considered by way of contrast. Next an estimate of the ratio of  $p_{N-2}$  to  $p_{N-1}$  is presented. This paper is principally about the modelling framework rather than statistical estimation, so this order-of-magnitude estimate will suffice; a more detailed statistical analysis will form a subsequent publication.

In GB, the proportion of all faults which are N-2 events may be as high as 20% (the precise value differs between DNO areas) [25]. For this upper limit of proportion of N-2 events, the rate of transition (either from N-0 or N-1) to N-2,  $\lambda_{N-2}$ , is thus  $\lambda_{N-1}/4$ . Taking mean times to restore customers by reconfiguration of 30 minutes<sup>2</sup> in N-2 and 15 minutes in N-1, which are typical for GB, then  $p_{N-2}/p_{N-1} = 1/2$ . As a consequence, calculations will be performed for a range of  $p_{N-2}/p_{N-1}$  up to this value.

The ‘small  $Y$ ’ results of the previous section will be used to demonstrate the dependence of the calculated capacity value on the choice of risk index, on the probabilities of each circuit state, and on the probability of a shortfall in the N-1 state.

### A. The Case Where the DG Cannot Run in the N-2 State

The case where the DG cannot contribute in the N-2 state, and hence  $Y_{N-2} = 0$ , will be considered first. This represents the situation in most present day GB demand groups, where distributed generators cannot alone provide frequency stability

<sup>2</sup>For the purposes of these reliability calculations we may without loss of generality take the mean restoration/repair time from N-1 to be 15 minutes. For N-1 events which do not disconnect customers the restoration time may be considerably longer, which gives a non-negligible number of transitions from N-1 (with all customers supplied) to N-2; however, the disconnections arising from these events are included in the treatment of the N-2 state.

in an entirely islanded demand group, and thus can only make a contribution to supporting demand in situations where there is an incoming circuit to provide frequency stability. While it is technically feasible to run an islanded microgrid, the necessary network upgrades have not yet widely been economically viable on the general network (as opposed to individual customers' sites).

1) *The case where single circuit capacity exceeds the maximum demand:* (i.e. obeys the P2/6 N-1 standard for a demand group without DG). The EPU conditional on the N-2 state must increase when the generation is added if the ELCC of  $Y$  is non-zero – however, there cannot be a corresponding decrease in the EPU conditional on the N-1 state, as it is already zero without the DG. As a consequence, the ELCC of  $Y$  must be zero. As will be discussed later, this illustrates the difficulty of extending a traditional deterministic N-x standard to include DG which is naturally modelled stochastically.

2) *MW EPNS, Small  $Y$ :* In this case, (9) reduces to

$$\begin{aligned}\nu_Y^{\text{ELCC}} &= \left( \frac{-p_{N-1} R'_D(c)}{p_{N-2} - p_{N-1} R'_D(c)} \right) \mu_{N-1} \\ &= \left( \frac{1}{1 + \frac{p_{N-2}}{p_{N-1} P(D > c)}} \right) \mu_{N-1},\end{aligned}\quad (19)$$

where  $\mu_{N-1}$  is the mean of  $Y$  conditional on the N-1 circuit state; the derivation notes that  $Y$  cannot contribute in the N-2 state and that there is no risk of supply shortage in the N-0 state. The ELCC is the mean of  $Y$  conditional on the N-1 state, scaled a factor which depends on the ratio of  $p_{N-2}/p_{N-1}$  to the probability that demand exceeds the single circuit capacity. As stated previously, in GB  $p_{N-2}/p_{N-1}$  may be as high as 0.5, so provided that the single circuit capacity is not much less than the maximum demand, in this case the ELCC will be small compared to  $\mu_{N-1}$ .

3) *EPNS Conditional on N-1 State, Small  $Y$ :* This case is mathematically equivalent to that studied in [23], and thus

$$\nu_Y^{\text{ELCC}} = \mu_{N-1} + \frac{R'_D(c)}{R_D(c)} \sigma_{N-1}^2. \quad (20)$$

In particular, in the limit of small  $Y$ , the ELCC is  $\mu_{N-1}$ .

4) *Percentage EPNS:* An alternative definition is to measure risk as EPNS divided by mean demand, i.e.

$$I^{\text{EPNS}'} = \frac{E[\max(D - X - Y, 0)]}{E[D]}. \quad (21)$$

The N-2 terms on each side of the equation defining the ELCC then cancel, and the ELCC is given by

$$(\mu_D + \nu_Y^{\text{ELCC}}) R_D(c) = \mu_D \int dy f_Y(y) R_D(c - \nu_Y^{\text{ELCC}} + y), \quad (22)$$

The small  $Y$  limit in this case is

$$\nu_Y^{\text{ELCC}} = \left( \frac{1}{1 + (R_D(c)/(R'_D(c)\mu_D))} \right) \mu_Y. \quad (23)$$

$R_D(c)/R'_D(c)$  is the characteristic decay scale of an exponential approximation to  $R_D(d)$  at  $d = c$ , will typically be much less than  $\mu_D$ . Hence in this case  $\nu_Y^{\text{ELCC}}$  will be close to the mean of  $Y$ .

As stated in the previous paragraph, the N-2 terms have cancelled on each side of (22), resulting in a substantially higher ELCC than using MW EPNS as the risk index. Indeed, (22) only differs from the case of EPNS conditional on the N-1 state by the appearance of  $\nu_Y^{\text{ELCC}}$  on its left hand side.

5) *Upper Bound on ELCC:* In the two circuit case, where the DG  $Y$  cannot support any demand in the N-2 case, the result (14) reduces to

$$\nu_Y^{\text{ELCC}} \leq \frac{p_{N-1}}{p_{N-2}} R_D(c), \quad (24)$$

where  $R_D(c)$  is the EPNS conditional on the N-1 circuit state. Typically  $R_D(c)$  will be small (the probability of a shortfall in the N-1 state will usually be low, and if there is a shortfall then this will be small), so if  $p_{N-1}/p_{N-2}$  is not much greater than 1 then this upper bound on  $\nu_Y^{\text{ELCC}}$  will be low also.

B. *Small  $Y$ : The Case Where DG Contributes in the N-2 State*

As with the case above where EPNS conditional on the N-1 circuit state is used as the reliability index, the case where  $Y$  is independent of the circuit state  $X$  is mathematically equivalent to that studied in [23]. In this case, for small  $Y$

$$\nu_Y^{\text{ELCC}} = \mu_Y + \frac{f_M(0)}{F_M(0)} \sigma_Y^2, \quad (25)$$

where the random variable  $M = X - D$ , i.e. the margin of exiting supply over demand. Once more, in this case the small- $Y$  limit of the ELCC is the mean of  $Y$ .

C. *LOLP used as risk index*

If the snapshot LOLP is used as the risk index instead of EPNS, then in the exposition above  $R_D(w)$  represents the probability that demand exceeds  $c$ . In the case where  $Y_{N-2} = 0$ , the LOLP in the conditional on the N-2 state is necessarily 1 whether or not DG is present. As a consequence, the equation defining the ELCC reduces to

$$P(D > c) = \int dy f_{Y_{N-1}}(y) P(D > c - \nu_Y^{\text{ELCC}} + y). \quad (26)$$

It is notable that this calculation ignores any consequences for the N-2 state of adding the DG (and the ELCC to demand), resulting in a calculation which is equivalent to one conditional on the N-1 state. This is natural given the risk index, as if the DG cannot support demand in the N-2 circuit state then its presence or absence makes no difference to the probability of a shortfall conditional on being in the N-2 circuit state.

D. *Discussion*

This section has explored various alternative routes to calculating the ELCC of the additional generation:

- Whether to use LOLP or EPNS as the reliability index.
- Whether to work with percentage or MW EPNS.
- Whether to perform a calculation conditional on a particular circuit state.

In transmission-level calculations, there is no analogy to the discrete nature of the circuit states and the dependence of the available additional capacity on this (at transmission level,

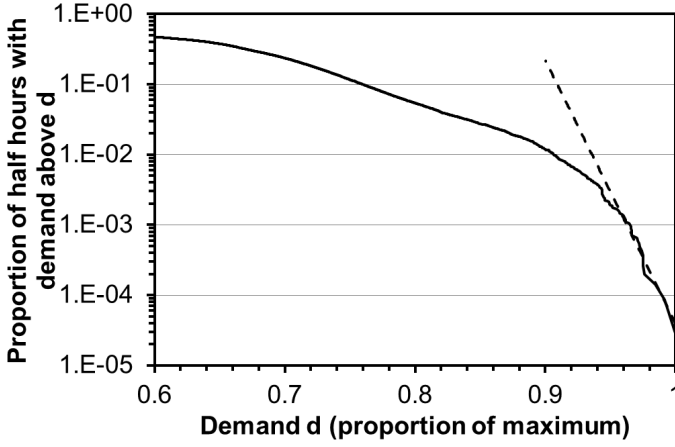


Fig. 2. Distribution near peak of metered demand on a feeder in North East England. The dashed line is a best fit exponential to the distribution above 95% of maximum demand (with decay constant 86.3 / [max demand])

even in small systems the number of generator states is much larger, meaning that typically  $X$  may be approximated reasonably well as a continuous random variable, and there is no physical mechanism to create a substantial dependence between  $X$  and  $Y$ .) Transmission ELCC results thus do not depend strongly on which options are taken in the various choices above. However, this discussion has shown how very different results are seen in this distribution level calculation depending on precisely how the calculation is performed. Great care must therefore be taken in ensuring that the chosen capacity value methodology does indeed represent properly important features of the engineering problem at hand. Consequences of this for practical planning situations will be discussed later, in Section VII.

## V. DATA FOR EXAMPLES

This section describes the data used for the sample results presented in the next section. The intention is to create test examples which are generally representative of data from typical demand groups in Great Britain, in order to understand how the calculated capacity value depends on the input probability distributions, rather than presenting a study of any one specific area of network in GB.

Fig. 2 summarises a year of demand data from a substation in NE England<sup>3</sup>; the proportion of half hour periods with demand above a given level is plotted. This is a smaller dataset than is ideal for estimation of a distribution of demand for risk modelling purposes. To give some degree of statistical smoothing, for calculations in this paper the distribution of demand is taken to be a least squares fit of an exponential function to the data above 95% of the metered peak demand<sup>4</sup>, giving

$$P(D > z) = e^{76.12 - 86.27z} \quad (27)$$

<sup>3</sup>The precise location of the demand group and wind farm cannot be revealed; they are chosen to be typical of data used in GB planning studies.

<sup>4</sup>Specifically, linear least squares regression is performed between the logarithm of the empirical distribution function of the demand data, and the demand level.

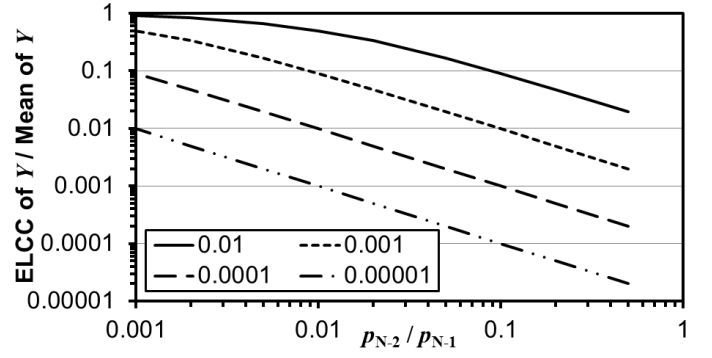


Fig. 3. Dependence of the small  $Y$  result for ELCC on the ratio of  $p_{N-1}$  to  $p_{N-2}$ , and on the LOLP conditional on the  $N-1$  state.

where demand is measured in units of historic measured peak. Parameterising the distribution of demand in this way can also provide a convenient means of considering sensitivity of the data to this distribution.

As many DG units in GB are wind farms, in order to demonstrate how calculation results depend on the form of the distribution of  $Y$  (which is very different for wind and conventional units), a year of metered wind output data from a wind farm in NE England is also available. The probabilities of different circuit states have already been discussed in Section IV to support discussion of that section's results.

## VI. EXAMPLE RESULTS

This section presents results for the case where the DG cannot run in an islanded demand group without incoming circuits to provide frequency stability, which represents the situation in most present GB distribution systems as described in Section IV-A. Application of this paper's methods to situations where (after a fault) incoming circuit capacity can be provided by network reconfiguration is discussed in Section VII-D.

### A. Small $Y$ approximation

Fig. 3 illustrates the dependence of the 'small  $Y$ ' result for ELCC on the ratio of the probabilities of the  $N-1$  and  $N-2$  circuit state, and on the LOLP in the  $N-1$  state  $P(D > c)$  where  $c$  is the single circuit capacity. Due to its simple form, one may conveniently (as is the case here) study the dependence of this expression on its inputs without reference to a specific scenario – however, for typical GB situations the ratio of the probabilities would be towards the right hand end of the horizontal axis, and the expected number of periods of shortfall would be at the very most a few hours per year (i.e. nearest to series for LOLP conditional on  $N-1$  of 0.00001).

Two notable features of this plot are that the ELCC as a proportion of the mean available capacity decreases as the ratio  $p_{N-2}/p_{N-1}$  increases, and also decreases as the LOLP conditional on  $N-1$  decreases. This is explained by the dependence of the small  $Y$  expression on  $p_{N-2}/(p_{N-1}P(D > c))$ , i.e. the probability of being in the  $N-1$  state (in which the DG  $Y$  can operate and support demand) conditional on a shortfall.

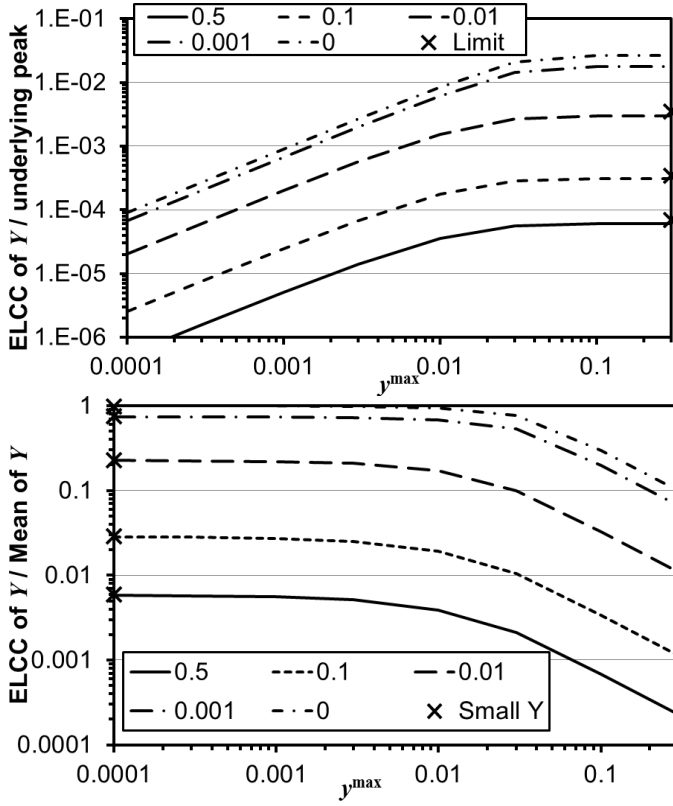


Fig. 4. Dependence of ELCC of a two-state unit  $Y$  of availability probability 0.9 on its installed capacity, and on the ratio  $p_{N-2}/p_{N-1}$  (by which series are labelled). The installed DG capacity  $y^{\max}$  is measured relative to the historic peak demand (in which 'Limit' refers to the result for the upper limit on the ELCC of  $Y$ ); Lower panel: ELCC as a proportion of the mean of  $Y$ .

### B. Two State DG Unit

Fig. 4 illustrates the dependence of the ELCC of a two state DG unit of availability probability 0.9 on its installed capacity and the ratio  $p_{N-2}/p_{N-1}$ . The single circuit capacity is taken as 95% of the historic peak demand, and the exponential approximation to the distribution of demand derived in Section V is used. The result for  $p_{N-2}/p_{N-1} = 0$  is equivalent to a result conditional on the  $N-1$  circuit state.

Once more, a notable feature of these results is the decrease in ELCC as the ratio  $p_{N-2}/p_{N-1}$  increases, i.e. as the  $N-2$  state comes to dominate the risk index. As is commonly seen with capacity value results, the calculated ELCC as a proportion of the mean of  $Y$  decreases as the installed capacity increases. The eventual saturation seen in the upper panel is explained by the fact that once the installed capacity of the two state unit is above a certain level its precise capacity makes little difference; if it is available, it reduces the risk in the  $N-1$  circuit state to a negligible level.

Particularly striking is how, when the  $N-2$  state (in which  $Y$  does not contribute) dominates the calculated risk, the ELCC remains very small even at large installed DG capacity. The upper bound for ELCC derived in III-B is actually quite a tight upper bound in this example (see black crosses in the upper panel of Fig. 4; the importance of this result however lies in the fact that this same upper bound applies to any

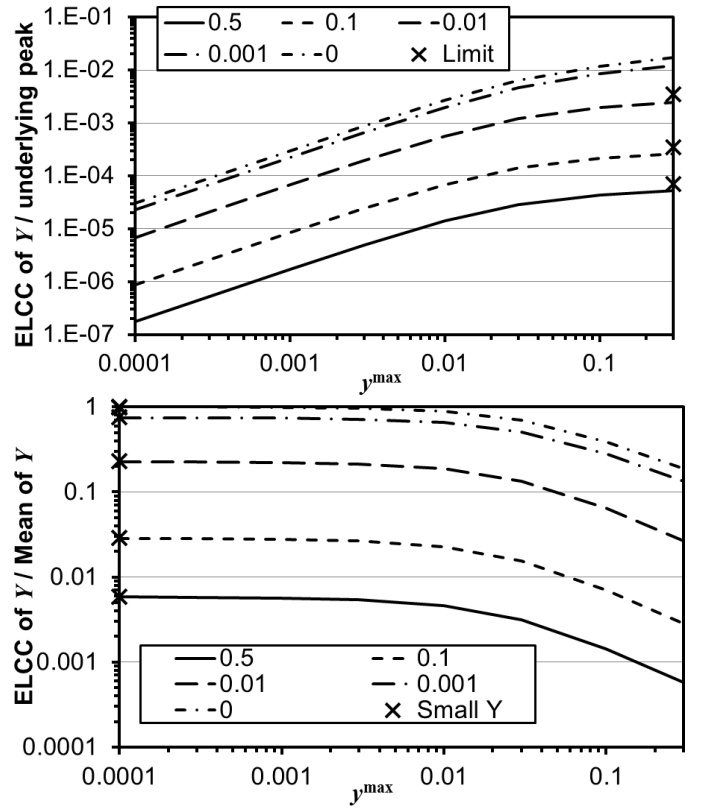


Fig. 5. Dependence of ELCC of a wind farm on its installed capacity, and on the ratio  $p_{N-2}/p_{N-1}$  (by which series are labelled). The installed wind capacity  $y^{\max}$  is measured relative to the historic peak demand. Upper panel: ELCC as a proportion of historic peak demand (in which 'Limit' refers to the result for the upper limit on the ELCC of  $Y$ ); Lower panel: ELCC as a proportion of the mean of  $Y$ .

DG unit or combination thereof, irrespective of the consequent distribution of available capacity  $Y$ .)

### C. Wind Farm

Fig. 5 illustrates the dependence of the ELCC of the wind farm on its installed capacity and the ratio  $p_{N-2}/p_{N-1}$ . The circuit and demand data are as for the two state unit example, and the empirical historical distribution of available capacity of the wind farm described in Section V is rescaled to the installed capacity stated; for estimation of the distribution of  $Y$  independence of  $D$  and  $Y$  is assumed conditional on being in the peak season. Again, the result for  $p_{N-2}/p_{N-1} = 0$  is equivalent to a result conditional on the  $N-1$  circuit state.

The general trends in these results are similar to those for the two state unit. The principal differences are that the ELCC of the wind farm is, taking all other data the same, lower than the ELCC of the two state unit due to the wind farm's lower mean output; and that the ELCC of the wind farm saturates less rapidly at large installed capacity, as for a unit whose available output can take any value between zero and installed capacity, even for a large installed capacity there is it does not the risk to negligible level when it is available.



## VII. DISCUSSION

### A. Defining ELCC

The key observation from the results above is that the calculated ELCC of DG in this framework depends strongly on the precise definition. Variouslly:

- If the risk index is EPNS, then if N-2 dominates the EPNS and the DG cannot contribute in N-2 then the ELCC will be very small.
- If the DG cannot contribute in N-2, then ELCC with respect to LOLP (not conditional on a specific circuit state) is equal to that conditional on the N-1 state.
- If EPNS divided by mean demand is used, then ELCC calculated using EPNS not conditional on any particular circuit state is very similar to that calculated using EPNS conditional on N-1.

This is different from transmission level calculations, where while it is important to use an appropriate capacity value definition, the choice of index (e.g. ELCC or EFC, and EPNS or LOLE) makes a much smaller difference to the result.

It is natural then to ask which is the best index to use. We do not offer a definitive answer to this, as this depends on the particular system planner's point of view – there are essentially two ways of looking at this problem, which give very different conclusions:

- 1) *If DG cannot contribute with no incoming circuits available, then this should be reflected in the calculated ELCC.* This perspective naturally is reflected by in an ELCC calculation with respect to a risk index not conditional on any particular circuit state.
- 2) *With no circuits available all the customers are lost whether we have DG or not, so nothing has changed there – and thus the risk index should be calculated with respect to the circuit state in which the present N-x standard is defined.* In the two circuit state above, this perspective would naturally be reflected by making the risk index conditional on the N-1 state.

If one is to perform a calculation conditional on the N-x state, then we believe that it is best to make this assumption explicit, rather than taking the options of performing a calculation using EPNS as a percentage of mean demand, or LOLP, which make that assumption implicitly. In addition, basing indices on EENS/EPNS is more relevant to cost-benefit analysis of reinforcement projects.

It is also important to note once more that if the risk of a capacity shortage in the N-1 state is zero, then adding DG cannot support more demand without increasing the risk level in that circuit state, and this should be considered when defining a capacity value for the DG. Under the definitions above, if the DG cannot contribute in worse circuit outage states, then its ELCC will then necessarily be its credible minimum available capacity. This illustrates the inevitable difficulties inherent in extending a legacy deterministic standard to include new resources which cannot naturally be treated deterministically. It may be that the only natural way to incorporate DG is to move to a full probabilistic standard. One should bear in mind that DG having a low capacity value would not mean that it is in any sense 'useless' – there are other ways in which it can

provide value to the system such as price arbitrage or voltage support.

### B. Length of Time Window Considered

In most areas of distribution network in Britain, the highest demands occur in central winter. Assuming in the two state example that circuit maintenance is not taken at a time when demand can exceed the N-1 capacity, then EENS conditional on the N-1 state is thus dominated by times in central winter, and is independent of the precise length of the time window over which the calculation is performed. However the risk conditional on the N-2 state is present year round, so that component of EENS does depend on the time window of the calculation. This is in contrast to most transmission level calculations, which are in this sense analogous to the 'conditional on N-1' calculation of this paper.

While this instability of results versus the time window chosen might seem undesirable, it in fact reflects the underlying nature of the problem, and hence in practical calculations one must be aware of it, and specify and interpret the calculations in an appropriate way to support the decision to be taken.

### C. Practical Planning Standards

One benefit of the present P2/6 standard is that it may be applied in practice by a wide range of engineers who are not necessarily specialist in probabilistic modelling; it does not require a full probabilistic risk assessment to be carried out for every planning decision. While a capacity value as defined in this paper might be very valuable in visualising DG's contribution within such full calculations, it is not certain that there will be a simplified formula which will reasonably accurately reproduce the ELCC of DG, because of the dependence of the calculated ELCC on all aspects of system background as well as the properties of the DG itself. Most notably, as in transmission level calculations, except for very small independent additional capacities, the ELCCs of multiple DG facilities will not be additive.

Despite this complication in wide implementation, we argue that any assumptions made in a practical planning model must be validated against a more detailed model and against a wide range of stress scenarios – otherwise the network will be exposed to uncontrolled risks.

### D. Relevance to More Complex Network Situations

The examples presented here have been for a relatively simple demand group with two incoming circuits and no low voltage interconnection. The same framework generalises to group with more incoming circuits, the same analytical results may then be used to understand the modelling results, and similar arguments will apply as to the choice of index and circuit state of which a standard should be based.

One simple generalisation of the two circuit example is to a network where not all customers can be restored by reconfiguration after an N-2 event. This adds an additional state, the post-reconfiguration pre-repair N-2 state which will be denoted N-2'. If the mean time to repair from N-2 is 12

hours, then  $p_{N-2'} = 24p_{N-2} = 12p_{N-1}$ . If the DG cannot contribute in N-2 even post-reconfiguration, then (for example) in the small  $Y$  the ELCC would be

$$\nu_Y^{\text{ELCC}} = \left( \frac{-p_{N-1}R'_D(c)}{\alpha p_{N-2'} + p_{N-2} - p_{N-1}R'_D(c)} \right) \mu_{N-1} \quad (28)$$

where  $\alpha$  is the proportion of customers who cannot be restored by reconfiguration after an N-2 event. It may be seen that where  $\alpha$  is non-zero,  $\nu_Y^{\text{ELCC}}$  is suppressed still further as compared to the  $\alpha = 0$  case in the main examples.

The model used does not account for network reliability within the demand group, which is consistent with the present GB standard; this approach will also prove valuable in interpreting results from more detailed calculations. The probability model might also be extended in a more formal way to include low voltage interconnection using standard methods for reliability of multi-area systems, though one might then need to take care regarding precise locations of DG and network faults when specifying parameters of the DG units.

It would further be possible to include within component reliability models more detailed consideration of generation connections, substation layout, etc. While this may reduce the opportunities for direct use of analytical results, the same general mathematical structure and intuition on principal drivers of results should still be relevant.

### E. Alternative Reliability Indices

This paper has discussed the definition of the capacity value of distributed generation, and consequent issues of calculation of these capacity values. The underlying risk indices used in the paper are the common expected value indices Expected Energy Not Supplied and Loss of Load Expectation; however, the same Effective Load Carrying Capability definition could be applied in combination with any other index which acts as a single summary of the overall adequacy risk in the demand group. While it is unlikely that equivalent analytical results will be available in the case of indices which can only be evaluated through a risk calculation which explicitly accounts for time correlations, the intuition as to drivers of risk calculation available from the calculations presented here will be relevant to interpretation of results based on alternative indices.

## VIII. CONCLUSIONS

This paper has discussed defining the capacity value of distributed generation, based on its contribution to reducing the need for incoming network capacity without changing adequacy risk levels. This is done in analogy with capacity value definitions which are widely used at transmission level. However, if the DG contribution depends on the circuit state then the at distribution level the result can depend very strongly on the precise calculation method. Notably if circuit states in which the DG cannot contribute dominate the reliability index (e.g. if the DG is unable to run in an islanded demand group without any incoming circuit capacity to provide frequency stability), then the calculated DG capacity value might be very small.

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